Subject

DESIGN OF AN EXPERIMENT TO STUDY $K^{\pm}_{e4}$ AND $K^{\pm}_{\mu 4}$ DECAYS
I. GENERAL CONSIDERATIONS AND TRIGGERING DESIGN

There now exist several studies of the K_{e4} mode, but no systematic studies of $K_{\mu 4}$. We consider here the design of a counter-spark chamber experiment to study both modes. A bubble chamber does not seem suitable to studying $K_{\mu 4}$ because of the very low abundance of the mode, which we will take as being about 3×10^{-6}.

I. GENERAL DESIGN OF A $K_{\mu 4}$ DETECTION EXPERIMENT

The major problem in studying the $K_{\mu 4}$ mode is the identification of the events. Not only is the desired mode infrequent; it is diluted by a flood of counterfeit events arising from the decay in flight of one of the pions from the abundant (5.6%) tau mode. The experimental design is primarily directed toward distinguishing real events from a very large spurious background.

Let us suppose that we trigger only on events in which three charged particles emerge: only one in 15 decays satisfies this condition. If we postulate a trigger requiring in addition that one of the three particles be a muon, an additional factor of the same order is gained. The gain is limited by the contamination of spurious events, from $\pi\mu$ decay in flight, of pions from the tau mode. It can be improved by raising the momentum of the decaying K, the improvement being directly proportional to the momentum: but then the accuracy of the measurement must be correspondingly increased to rule out the residual contamination. We will consider only K's in the momentum range available from the ZGS: higher momenta will not be available in this country for several years.

Exclusion of the spurious tau mode cannot be achieved by any trigger depending on particle identification; it must come from detecting the pion decay in flight. This can be done by scanning: i. e. direct observation of the kink indicating a pi-mu decay; or by measuring, i. e. demonstrating that the kinematics of the decay is that of a tau rather than that of a $K_{\mu 4}$. It is preferable to use scanning, which is probably quicker and cheaper; but there will inevitably be a large residue for which measuring is necessary. Also, as we will see later, there is a fraction of $K_{\mu 4}$ decays which are completely indistinguishable from a tau in which pi-mu decay occurs very close to the K-decay point.

Magnitude of the background. Since the total triggering rate will not be much less than 1/200 of the total decay rate, there will be about 2000 triggers per true event; plus the inevitable "junk" triggers that usually increase the total by anything from 20% to a factor of 2 or more. We must consequently be prepared to record anywhere from two to four thousand pictures for every real event. The "junk" background triggers are readily eliminated by scanning, as are those events in which the pi-mu kink is visible. The fraction of events in which the pi-mu kink is visible to a scanner depends critically on momentum and on the experimental arrangement, but for geometrical reasons is unlikely in any case to exceed 90%. Thus we must be prepared to measure anywhere up to 500 spurious events to find one true event. Such a measuring program is conceivable with automatic or semi-automatic measuring systems like CHLOE or POLLY; either of these could readily tackle the job, since measuring very large numbers of such simple events is exactly what they are good at.

II. ELIMINATION OF THE TAU MODE BY KINEMATICS

We must now demonstrate that measuring the background of tau decays can yield a sample of $K_{\mu 4}$ decays sufficiently free from contamination. We note first that there exists a small region kinematically excluded for taus that is accessible to $K_{\mu 4}$; e.g. the maximum c.m. momentum in tau decay is 126 Mev/c, in $K_{\mu 4}$ 151. In addition, the accurate determination of particle directions and momenta in the decay permits considerably more discrimination. First, machine measurement will pick up additional decays in flight that escape detection in scanning. Next, we note that observation of the direction and momentum of the two particles identified as pions (the third is the muon) allows a one-constraint fit to the event; the momentum and direction of the missing pion are then predicted. If the missing mass is calculated, it must be that of the pion for a tau-decay; for a $K_{\mu 4}$ the missing mass will lie between 105 and 225 Mev. However, it appears that this is not a very sensitive criterion, in view of the relatively poor determinati on of missing mass accorded by conventional measurement accuracy. However, if the missing particle turns out to have the <u>direction</u> predicted for the missing pion, then it can be a $K_{\mu 4}$ decay only if the unobserved neutrino momentum lies in the same direction (though not necessarily with the same sense). In this case the missing mass of the dilepton <u>cannot</u>

be that of the pion; there must be a discrepancy of at least 28 Mev, the momentum of the neutrino in pi-mu decay. It is not possible for a muon-neutrino combination to exhibit the same invariant mass as a pion and at the same time yield a muon with the same direction and momentum as the pion. Thus in general a distinction is possible.

The most difficult and subtle counterfeit is that in which the pion decays in the first millimeter or two of its path, so that its track is too short to allow detection of a kink. In this case the invariant mass of the dilepton is again that of the pion. This background can be evaluated by comparing with the number of events in which the pi-mu decay occurs after a centimeter or so, in the next few centimeters. The number of pi-mu decays in the first 5 mm, say, depends on the mean pion momentum. For 3-Bev/c K'S, the mean pion momentum is 1 Bev/c, and the probability of decay in the first 5 mm is 1×10^{-4}. With 3 pions, the total probability of this event is $5 \times 10^{-2} \cdot 3 \times 10^{-4} = 15 \times 10^{-6}$, or about 5 times the true rate. These will form a huge bump in the dilepton mass spectrum, and must be subtracted; it may be necessary to discard data in this portion of the spectrum.

The probability of the ultimate success of this kind of endeavor cannot be evaluated without a careful and extensive set of Monte Carlo calculations, in which both the true and background tau events are generated, suitable measurement errors postulated, and analysis made to see how successful the removal of the tau contamination can be.

EXPERIMENTAL ARRANGEMENT FOR $K_{\mu 4}$ DECAY

The experimental system requires a clear pi-mu distinction over a reasonable fraction of the momentum range; this, together with the improved signal to noise ratio at high momentum values, points to the use of decay in flight at somewhere between 2 and 5 Gev/c, the exact value depending on rates, backgrounds, and the available magnets for momentum determination.

The obvious mu identification procedure is by range, with enough (more is not useful because of the large τ background) absorber to attenuate pions by a factor of perhaps 300. This puts a lower limit on the momentum of the muon that can be detected, at somewhere around 500 Mev/c. For reasonable detection efficiency, this should be at or below the median muon momentum in $K_{\mu 4}$ decay so the lower limit of K momentum is around 1.5 Gev/c. Higher K momentum increases the detection efficiency not only by rising the muon momentum, but also by projecting all the particles into a narrower cone in the lab system.

Since the detection of pion decay in flight is so important, it is necessary to observe all or most of the trajectory of each particle; sampling of the trajectory, as e.g. with wire chambers, is not as satisfactory. Also, those decays in which the muon direction is nearly that of the pion, but there is a change of momentum, cannot be readily detected unless the momentum is observed both before and after the decay. (We have already discussed very short pion paths). Consequently, a magnetic field over the entire trajectory is desirable. Finally, it appears highly desirable to have not only good resolution and high accuracy of momentum and angle determination, but also good depth discrimination in order to observe kinks in any plane. All these considerations point toward a system of wide-gap chambers used in the spark mode; this has the best available resolution and accuracy in all dimensions. The limited angular sensitivity ($\pm$ 50°) is no handicap since nearly all particles are forward (99% within 25° or less.)

The experimental arrangement implied by these considerations is sketched, in the barest essentials only, in Fig. 1.

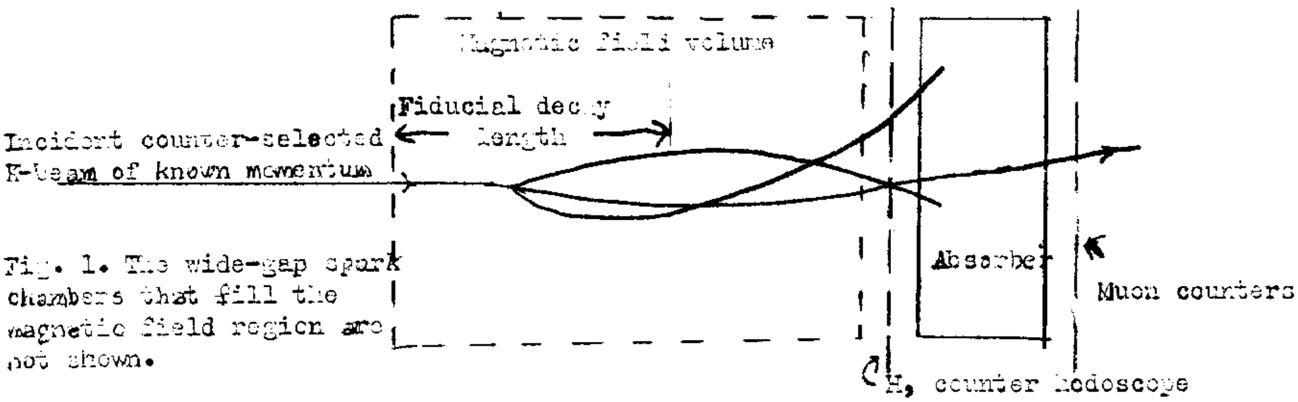


Fig. 1. The wide-gap spark chambers that fill the magnetic field region are not shown.

III. BEAM REQUIREMENTS AND DATA RATE: CHAMBER MEMORY TIME

The K momentum is determined by the conflicting requirements of a low value to increase the decay rate per unit path for a given flux, and to improve the absolute momentum accuracies; and a high value to improve signal-to-noise ratio and collimate the decay products into a narrow forward cone for efficient detection. As a reasonable compromise we take as a nominal value 3 Bev/c, which gives a small forward cone, a good K flux (1-2 x 10^4/pulse), the possibility of enrichment, and the possibility of using several existing magnets (including SCM 103, 104, 105 and the large freon bubble chamber magnet).

Use of unseparated beams. The limitation on the use of an unseparated beam is the cluttering of the spark chambers with random beam particles arriving within the memory time of the chamber. In the case of wide-gap chambers used in the spark mode, a distinction must be made between the single-event memory time and multiple-event retention, which brings in the phenomenon of erasure. If a wide-gap chamber is triggered on a single event, the tracks will still be visible if the high voltage is applied up to 2 to 5 microseconds later, depending on the clearing mechanisms used. However, if during that delay, a second event were to occur, it would have the effect of partially or completely erasing the previous one; the sparks preferentially form along the newest tracks. Thus the significant limitation on the allowable beam flux is the probability that an accidental beam particle traverse the system in the interval between the desired event and the time of high-voltage application; this time usually runs 0.4-0.8 sec. Thus, for purposes of beam calculations, we will suppose an event killed if a beam particle unluckily traverses the system during the high-voltage delay time; in actual fact, the onset of the period in which the new track is fatal may be somewhat delayed, nor is the mortality necessarily 100%. Because of this property, the the system will "paralyze" at high rates, and there will be a maximum event rate.

If we call the high-voltage delay time τ the probability that no count occurs in τ from a random source of average counting rate a is $\exp(-a\tau)$. Then the true event rate will be proportional not to a, but to $a \cdot \exp(-a\tau)$, which has a flat maximum at $a\tau = 1$.

Beam Intensities. Without enrichment the $K^{\pm}$/ (total plus beam) ratio at the decay region, for momenta near 3 Bev/c, will probably not exceed 5×10^{-3}. We can then make the following table of event rates for 0.5 sec. beam pulses, for both unseparated and enriched beams.

A. Unseparated beams.

Total No. of particles/pulse	No. of 3 Bev/c K^+ per pulse	No. of K^+ decays/meter/pulse	No. of taus with pi-mu decay/pulse	No. of true $K_{\mu 4}$ decays/pulse	Actual $K_{\mu 4}$ event rate with 0.5 μsec. delay
10^5	500	22.5	0.125	7×10^{-5}	6.3×10^{-5}
10^6	5000	225.	1.25	7×10^{-4}	4.2×10^{-4}
$2 \cdot 10^6$	10000	450	2.5	1.4×10^{-3}	4.8×10^{-4}

B. Separated Beams; assume an enrichment of 20:1

10^5	10000	450	2.5	1.4×10^{-3}	1.3×10^{-3}
$3 \cdot 10^5$	30000	1350	7.5	4.1×10^{-3}	3.5×10^{-3}

A rate of at least 10^4 K^+/ pulse is reasonable for the ZGS. The importance of enrichment is apparent; it would even be worth while to compromise somewhat on beam momentum and accept a lower value, in order to obtain enrichment, if this were necessary.

Since further discussion of the feasibility of a $K_{\mu 4}$ detection experiment must await extended Monte Carlo calculations, let us turn our attention to the K_{e4} mode to see whether the two leptonic four-body modes can be studied compatibly.

IV. KINEMATICS OF K_{e4} DECAY IN FLIGHT

The negligible mass of the electron makes the K_{e4} mode kinematically more distinct from the tau mode than is the $K_{\mu 4}$. The maximum momentum of a decay particle is 204 Mev/c, and the angular distribution of the electrons as well as their momentum spectrum differ markedly from that of the pions. For 3-Bev/c K-decay in flight into 3 charged particles, any particle emitted at an angle of 14° or more must be an electron. We also note that at 3 Bev/c any particle with momentum under 360 Mev/c is an electron; this includes over half the electron spectrum. The emitted particle spectra are about as follows: (based on a Monte Carlo sample of 100 events)

K-momentum	Pion momentum			Electron mom (Mev/c)			Angular distribution
	Min.	Mean	Max	Min.	Mean	Max.	
2 Bev/c	240 Mev/c	600	400	0	200	920	Pi-50% in 6°, 85% in 12° E -50% in 17°, 76% in 27°
3 Bev/c	360	960	2120	0	280	1440	Pi-69% in 6°, 92% in 9° E -55% in 12°, 81% in 21°

We must be prepared to accept wider angles and lower momentum particles than in $K_{\mu 4}$ decay; nor will we have to sacrifice the lower momentum portion of the spectrum in order to identify the lepton. No longer do we have the enormous flood of simulated events from tau decay. We do have to provide identification of the particle as an electron to obtain a suitable trigger. If we require that there be three charged decay products, one of which is an electron, we already achieve a far better signal-to-noise ratio than can be obtained with $K_{\mu 4}$.

Other decay modes giving three charged particles, including (at least) one electron, arise from internal conversion as follows:

Mode	Branching ratio		
$\pi^{+}\ \pi^{o} \hookrightarrow e^{+} + e^{-} + \gamma$	0.21 x 1/80	=	2.6×10^{-3}
$\pi^{o}\ e\ \nu \hookrightarrow e^{+} + e^{-} + \gamma$	0.048 x 1/80	=	0.6×10^{-3}
$\pi^{+}\ \pi^{o}\ \pi^{o} \hookrightarrow e^{+} + e^{-} + \gamma$	0.017 x 1/40	=	0.4×10^{-3}
$\pi^{o}\ \mu\ \nu \hookrightarrow e^{+} + e^{-} + \gamma$	0.034 x 1/80	=	0.4×10^{-3}

This gives a total background rate of 4.0×10^{-3}.

External conversion of gamma-rays in $\sim$0.01 radiation length of chamber plates and gas will contribute additional background as follows:

$\pi^{+}\ \pi^{o}$:	0.21 x .02	=	4.2×10^{-3}
$\pi^{+}\ e\nu$:	.048 x .02	=	1.0
$\pi^{+}\pi^{o}\pi^{o}$:	.017 x .04	=	.7
$\pi^{o}\ \mu\nu$:	.034 x .02	=	.7
			6.6×10^{-3}

Conversion in the hodoscope counters only gives one additional counter trigger. The total background rate is then about 1.0×10^{-2}, or 250 times the true K_{e4} rate. The background rate can be reduced by adding more triggering restrictions. Every background mode has at least two electrons and one gamma ray: so a gamma-ray anticoincidence counter and/or identification of a second electron would reduce the spurious trigger rate.

Thus a first approximation to a K_{e4} logic would add to the $K_{\mu4}$ logic some counters to catch particles emitted at larger angles or with lower momenta; and in the downstream region, a set of electron-identifying shower counters.

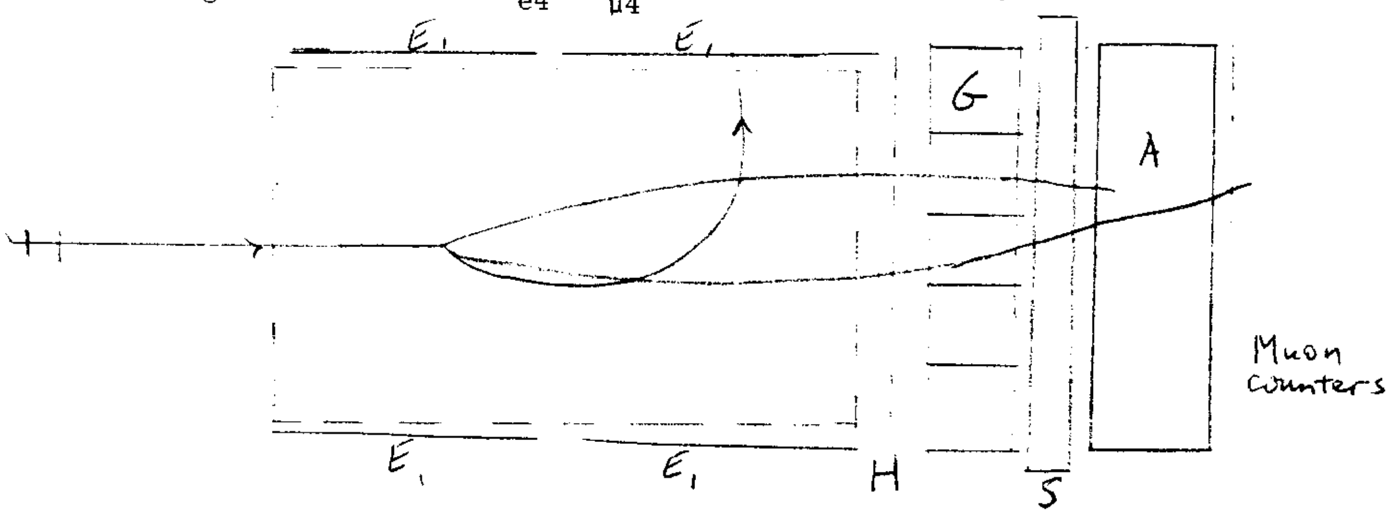
Event and background rates. If we assume the enriched beam of 10^4 K's described on page 6, we find that the total number of background triggers (not including junk) is 4.5 per pulse, and the true K_{e4} decay rate is 1.8×10^{-2}, or 18 perhour. Further background reduction by a factor of the order of 10, and a net useful event approaching 5-10 per hour then appears reasonable.

V. COMBINED EXPERIMENTAL ARRANGEMENT TO DETECT BOTH LEPTONIC MODES.

The K_{e4} detection system, since it allows more restrictive logic, is more complex than the $K_{\mu 4}$; and consequently we can design economically either a single-purpose simple $K_{\mu 4}$ detector, or a combined $K_{\mu 4}$-K_{e4} detector; the addition of the $K_{\mu 4}$ logic to the K_{e4} adds little complication other than the increase in picture taking rate. The use of parallel logical selection systems has been experimentally shown to work satisfactorily; each frame is labelled by the triggering logic used.

A schematic of the combined system is shown in Fig. 2.

Fig. 2. COMBINED K_{e4} - $K_{\mu 4}$ DETECTION SYSTEM.



As compared to Fig. 1, we have added the counters E_1 on the sides to catch low-momentum particles; and inserted between the counter hodoscope H and the muon filter absorber A, the shower detectors G, (plus, optionally, the spark chamber module S).

Each section of G is an independent lead-scintillator sandwich, biased to count only showers. A G pulse in coincidence with the corresponding H counter denotes an electron; without H it counts as a gamma-ray.

An acceptable K_{e4} trigger is then either (1) one E_1 counter, two H counters, and no counts in G; or (2) no E_1 count, 3 counts in H, one electron count in G, and no gamma counts in G. There will be some cases in which showers spread across adjacent G counters; if we allow these as acceptable triggers, the spark chamber S helps to identify good events. Alternatively, more complex G structures such as crossed arrays will be still more selective.

The array sketched above will probably reduce the K_{e4} background triggering rate by a factor between 3 and 10. We have not included electron-identifying counters for the low-momentum E_1 counters; conceivably unexpected background effects might make this desirable.

As with the $K_{\mu 4}$ decay, it is clear that extended Monte Carlo calculations will be required to give a more exact picture of the fraction of K_{e4} decays detected in a particular setup. Further study is also required to see just how the additional data on both decay modes contributes to new physics; that subject has not been considered here.

If we assume an enriched beam with 10^4 K's per pulse (as on p. 10), then the $K_{\mu 4}$ trigger rate is 2.5/pulse and the K_{e4} rate is 0.5/pulse; if we add about 50% for junk triggers, we will expect about 4.5 - 5 pictures per pulse, each labelled by the logic producing it.

Existing cameras will handle this rate easily. We then expect the following total rates:

	Per hour	Per day
Total pictures	5000	100,000
$K_{\mu 4}$ events	0.6	12
K_{e4} events	10	200

The $K_{\mu 4}$ rate has been reduced to take into account the loss of the low momentum end of the muon spectrum.

VI. FINAL REMARKS

The K_{e4} seems so much easier and more straightforward than $K_{\mu 4}$ that further consideration of the tactics of an initial experiment may be in order, especially after more detailed considerations that take into account the available magnets.